Evaluate $\int_0^1 z^2 \ln z \, dz$.

SCORE: _____ / 7 PTS

$$= \lim_{N\to 0^+} \int_N^1 z^2 \ln z \, dz \quad (1)$$

$$= \lim_{N\to 0^+} \left(\tfrac{1}{3}z^3 \ln z - \tfrac{1}{9}z^3\right)\Big|_N^1 \quad (2)$$

$$= \lim_{N\to 0^+} \left(-\tfrac{1}{9} - \tfrac{1}{3}N^3 \ln N + \tfrac{1}{9}N^3\right) \quad (1)$$

$$= -\tfrac{1}{9} - \tfrac{1}{3}0 + 0$$

$$= -\tfrac{1}{9} \quad (1)$$

$$\lim_{N\to 0^+} N^3 \ln N$$

$$= \lim_{N\to 0^+} \frac{\ln N}{N^{-3}} \quad (1)$$

$$= \lim_{N\to 0^+} \frac{N^{-1}}{-3N^{-4}}$$

$$= \lim_{N\to 0^+} -\tfrac{1}{3}N^3 \quad (1)$$

$$= 0$$

Evaluate $\int \frac{40-x^2}{x^3-4x^2+8x}\,dx$. $= \int \frac{40-x^2}{x(x^2-4x+8)}\,dx = \int \frac{40-x^2}{x((x-2)^2+4)}\,dx$ **SCORE: ____ / 8 PTS**

$$= \int \left(\frac{A}{x} + \frac{B(2x-4)+C(2)}{(x-2)^2+4} \right) dx \quad ①$$

$$① \quad 40-x^2 = A[(x-2)^2+4] + B(2x-4)x + C(2x)$$

$x=0$: $40 = 8A \rightarrow A=5$

$x=2$: $36 = 4(5)+4C \rightarrow C=4$

COEF x^2: $-1 = 5+2B \rightarrow B=-3$

CHECK $x=3$: $\frac{40-9}{27-36+24} \stackrel{?}{=} \frac{5}{3} + \frac{-3(2)+4(2)}{5}$

$① \quad \frac{31}{15} \stackrel{?}{=} \frac{5}{3} + \frac{2}{5} = \frac{25+6}{15}$ ✓

$$= \int \left(\overbrace{\frac{5}{x}}^{①} + \frac{\overbrace{-3(2x-4)}^{①}\overbrace{+4(2)}^{①}}{(x-2)^2+4} \right) dx$$

$$= \underbrace{5\ln|x|}_{\frac{1}{2}} \underbrace{-3\ln(x^2-4x+8)}_{\frac{1}{2}} \underbrace{+4\tan^{-1}\frac{x-2}{2}}_{①} + C$$

$\left(-\frac{1}{2}\right)$ IF YOU FORGOT

Evaluate $\int \frac{dt}{(1+e^t)^2}$. $= \int \frac{1}{u^2(u-1)}\,du = \int \left(\frac{A}{u} + \frac{B}{u^2} + \frac{C}{u-1}\right) du$ $\frac{1}{2}$

SCORE: ____ / 8 PTS

1 $u = 1+e^t$

$t = \ln(u-1)$

$dt = \frac{1}{u-1}\,du$

$1 = Au(u-1) + B(u-1) + Cu^2$ $\frac{1}{2}$

$u=0: 1 = -B \rightarrow B = -1$

$u=1: 1 = C$

COEF OF u^2: $0 = A + 1 \rightarrow A = -1$

CHECK $u=2$: $\frac{1}{4} \stackrel{?}{=} \frac{-1}{2} + \frac{-1}{4} + 1$ ✓ $\frac{1}{2}$

$= \int \left(\frac{-1}{u} + \frac{-1}{u^2} + \frac{1}{u-1}\right) du$ (1) (1) (1)

$= -\ln|u| + \frac{1}{u} + \ln|u-1| + C$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$= -\ln(1+e^t) + \frac{1}{1+e^t} + t + C$ (1)

$-\frac{1}{2}$ IF YOU FORGOT

☆ SEE ALTERNATE SOLUTION BELOW IF YOU USED $u = e^t$

☆ TALK TO ME IF YOU USED ANOTHER METHOD

$$\int \frac{dt}{(1+e^t)^2} = \int \frac{du}{u(1+u)^2} = \int \left(\frac{A}{u} + \frac{B}{1+u} + \frac{C}{(1+u)^2} \right) du \quad \frac{1}{2}$$

$\frac{1}{2}$ $\quad$ 1

$u = e^t$

$t = \ln u$

$dt = \frac{1}{u} du$

$$1 = A(1+u)^2 + Bu(1+u) + Cu \quad \frac{1}{2}$$

$u=0: \ 1 = A$

$u=-1: \ 1 = -C \longrightarrow C = -1$

COEF OF u^2 : $0 = 1 + B \longrightarrow B = -1$

CHECK $u=-2$: $\frac{1}{-2} \overset{?}{=} \frac{1}{-2} + \frac{-1}{-1} + \frac{-1}{1}$ ✓ $\frac{1}{2}$

$1 \quad 1 \quad 1$

$$= \int \left(\frac{1}{u} + \frac{-1}{1+u} + \frac{-1}{(1+u)^2} \right) du$$

$$= \frac{1}{2} \ln|u| - \frac{1}{2} \ln|1+u| + \frac{1}{1+u} \frac{1}{2} + C$$

$$= t - \ln(1+e^t) + \frac{1}{1+e^t} + C$$

1

$-\frac{1}{2}$ IF YOU FORGOT

Evaluate $\int_0^{\infty} \frac{dy}{y(\ln y)^2}$. $= \int_0^{\frac{1}{e}} \frac{dy}{y(\ln y)^2} + \int_{\frac{1}{e}}^{1} \frac{dy}{y(\ln y)^2} + \int_1^{e} \frac{dy}{y(\ln y)^2} + \int_e^{\infty} \frac{dy}{y(\ln y)^2}$ 2

SCORE: _____ / 7 PTS

DIVERGES 1

$u = \ln y$

$du = \frac{1}{y} dy$

$\int \frac{1}{u^2} du = -\frac{1}{u} = -\frac{1}{\ln y}$

☆ TALK TO ME IF YOU EVALUATED $\int_{\frac{1}{e}}^{1} \frac{dy}{y(\ln y)^2}$ INSTEAD

NOTE: OTHER 2 INTEGRALS CONVERGED

$\lim_{N \to 1^+} \int_N^e \frac{dy}{y(\ln y)^2}$ 1

$= \lim_{N \to 1^+} -\frac{1}{\ln y} \Big|_N^e$ 1

$= \lim_{N \to 1^+} \left(-1 + \frac{1}{\ln N}\right) \to \infty$ 1 1